PAIRWISE INTERACTION OF DROPS OF MAGNETIC EMULSION UNDER THE ACTION OF ROTATING FIELD

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UDC 532.5:538.6

Consideration is given to the dynamics of a pair of drops of a magnetic liquid that are suspended in a normal liquid with relatively low viscosity under the action of a rotating field. Account is taken of hydrodynamic and dipole-dipole interactions as well as of the electrodynamic torque due to the finite relaxation time of magnetization of the magnetic liquid. It is shown that hydrodynamic interaction leads to the formation of a rigid pair in the rotating field and the electrodynamic moment of forces governs its dynamics at large frequencies, leading to a growth that is linear in the field frequency in the rotational velocity for the pair of drops, whereas a pair of solid particles loses sensitivity to the field.

The practical interest in polarizable and magnetizable liquid-dispersed media is due to the possibility of controlling their properties and transfer processes in them by the action of external fields. The electric or magnetic interaction of particles of a dispersed phase serves, in many cases, as the main channel of the influence of the field. Computer modeling of even comparatively small ensembles of particles enables us to obtain useful information about the character of processes in the dispersion structure that are induced by the field and about the influence of these processes [1]. The simplest case that permits a most accurate and complete investigation is that of pairwise interaction in a rotating field, which has been considered experimentally [2, 3] for nonmagnetic particles in a magnetic liquid and theoretically [4, 5] for soft-magnetic particles in a normal liquid.

In recent years, suspensions of an encapsulated magnetic liquid (colloid of nano-dimensional ferromagnetic particles) [6] as well as magnetoliquid emulsions [7] have been investigated along with traditional dispersions of solid particles. The pairwise interaction in these media under the action of a rotating field is considered in this work.

Let the viscosity η_0 of the magnetic liquid in the drop be higher than the viscosity η of the ambient medium. This enables us to exclude from consideration any motion of the liquid in the drops that is different from quasisolid motion.

The equilibrium magnetization of a magnetic liquid in a field of strength H is described by the Langevin law:

$$M_0 = M_s L(\xi) , \quad \xi = m_0 H / kT , \quad L = \operatorname{cth} \xi - 1/\xi .$$

where M_s is the saturation magnetization; m_0 is the magnetic moment of the colloidal ferroparticle; k is the Boltzmann constant; T is the absolute temperature.

The rate of magnetization relaxation is determined by the characteristic time of Brownian rotational diffusion of the colloidal particle

$$\tau_r = 3 v \eta_0 / kT ,$$

where v is its "hydrodynamic" volume. Usually, $\tau_r < 10^{-4}$ sec.

Academic Scientific Complex "A. V. Luikov Institute of Heat and Mass Transfer of the Academy of Sciences of Belarus," Minsk, Belarus. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 69, No. 1, pp. 98-102, January-February, 1996. Original article submitted November 18, 1994.

 1062-0125/96/6901-0084$15.00

Let the rotational frequency of the field ω_0 be low as compared to the inverse time of magnetic relaxation ($\omega_0\tau_r << 1$). Then the deviation of the magnetization from equilibrium is also small and is described by the linear relation [8]

$$\mathbf{M} = \left(M_0 - \tau_{\parallel} \frac{dM_0}{dt}\right) \mathbf{h} - \tau_{\perp} M_0 \left(\frac{d\mathbf{h}}{dt} - \mathbf{\Omega} \times \mathbf{h}\right), \tag{1}$$

where $\tau_{\parallel}$, $\tau_{\perp}$ are the relaxation times; $\mathbf{h}$ is a unit vector in the direction of the field; $\mathbf{\Omega}$ is the hydrodynamic vorticity of the magnetic liquid. In the approximation in question it coincides with the rotational velocity of the drop.

Because of the deviation from equilibrium the electrodynamic torque

$$\mathbf{T} = V\mathbf{M} \times \mathbf{H},$$

acts on the drop (V is the volume of the drop).

According to [4], for a pair of solid particles, the plane of rotation of the field is attracting. It can be shown that this property also holds for a pair of drops. When considering plane motion ($\mathbf{\Omega}\times\mathbf{H} = 0$) we have in accordance with (1)

$$\mathbf{T} = 4\eta_r V (\omega_0 - \mathbf{\Omega}). \tag{2}$$

The torque T is proportional to the velocity of rotation of the field about the drop, its volume, and the rotational viscosity of the magnetic liquid $\eta_r = M_0 H \tau_{\perp}/4$. The latter characterizes the energy dissipation in ordered rotation of ferrocolloid particles in a viscous medium and can be determined by the relation [8]

$$\eta_r = \frac{3}{2} \varphi_h \eta_0 \alpha^* L^2(\xi)\, \xi\, (2\xi + \alpha^* L(\xi)\, (2 + L(\xi)\, \xi)^{-1},$$

where φ_h is the hydrodynamic concentration of ferroparticles; α^* is the phenomenological parameter of their effective magnetic anisotropy. For typical magnetite-based magnetic liquids, α^* changes within the limits of 1–4 [8], and for a material with a larger value of the constant K of magnetic anisotropy, $\alpha^* = KV_m/(2kT)$.

The rotational velocity of an isolated drop is determined from the condition of balance of the electrodynamic moment of forces (2) and the moment of viscous friction forces $-6V\eta\mathbf{\Omega}$ as

$$\mathbf{\Omega} = \omega_0 2S/(3 + 2S), \quad S(\xi) = \eta_r(\xi)/\eta.$$

According to this relation, the rotational velocity of the drop is proportional to the rotational velocity of the field and depends on the ratio of the rotational viscosity of the magnetic liquid to the viscosity of the ambient liquid S. Within the limit $S \to \infty$, the drop rotates with the frequency of the field.

There are magnetic and hydrodynamic interactions between two rotating drops. In a dipole approximation, the magnetic force with which the first drop acts on the second is

$$\mathbf{F} = 3m^2 R^{-4} \left(\mathbf{n} + 2\mathbf{h}\,(\mathbf{nh}) - 5\mathbf{n}\,(\mathbf{nh})^2\right). \tag{3}$$

Here $m = VM$ is the magnetic moment of the drop; $\mathbf{n}$ is a unit vector in the direction from the center of the first drop to the center of the second drop; R is the distance between the centers. The force $-\mathbf{F}$ acts on the first drop.

The vector of the hydrodynamic interaction force of the rotating drops given that the Reynolds number is small is a linear function of the pseudovector of the angular velocity of their rotation. The only possible form of this dependence is $f(R)\mathbf{n}\times\mathbf{\Omega}$. Consequently, hydrodynamic interaction does not affect drop motion along the line of the centers and for them the result obtained in [5] for solid particles holds true. According to [5], the growth in viscous friction when particles with an ideally smooth surface approach each other leads to the formation of a rigid pair in the rotating field. It is precisely the rigid pair that we consider next. The relative motion of drops is absent in this pair, and its behavior is completely described by the rotational velocity $\mathbf{\Omega}$. The value of $\mathbf{\Omega}$ is determined from the equation of balance of the viscous, dipole, and electrodynamic moments of forces

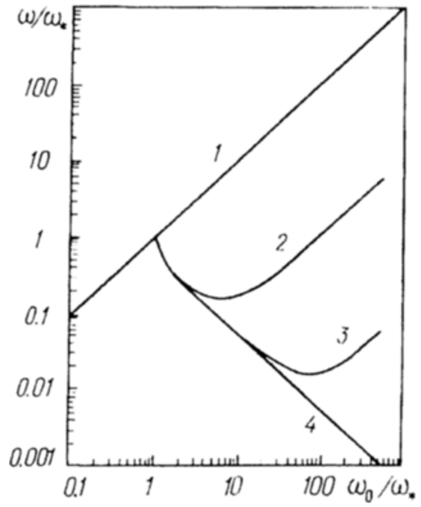

Fig. 1. Rotational frequency of pair of magnetic drops suspended in a liquid ω vs rotational frequency of field ω_0 (ω_* is the critical synchronism frequency) for $Q = 1$ (1), 10^{-2} (2), 10^{-3} (3), and 0 (4).

$$6\eta V\Omega = C\,(a\mathbf{n} \times \mathbf{F} + \mathbf{T})\,, \tag{4}$$

where C is a dimensionless form factor that characterizes the viscous friction of the rotating pair as compared with the friction of a rotating sphere, and **F** and **T** are determined in (2) and (3). For the angle φ of pair orientation, we have

$$d\varphi/dt = \omega_*\,(1 - Q)\sin 2\,(\omega_0 t - \varphi) + Q\omega_0\,, \tag{5}$$

where

$$\omega_* = 0.402\pi M_s^2 L^2\,(\xi)/18\eta\,, \quad Q = S\,(\xi)\,(2.8 + S\,(\xi))^{-1}\,.$$

The value of $C = 0.534$ is found from a comparison with the result [5] for particles.

According to (5), up to the frequency $\omega_0 = \omega_*$ the pair of drops rotates in synchronism with the field, lagging behind by the angle (1/2)arcsin (ω_0/ω_*). In the following, the rotation is of a step character and, for its average velocity, from (5) we obtain

$$\omega = \omega_0 - (1 - Q)\sqrt{\omega_0^2 - \omega_*^2}\,. \tag{6}$$

In the case of $Q = 0$, which means equality to zero of the rotational viscosity of the magnetic liquid, relation (6) yields the result [5] for solid particles. The main difference between the drops and the particles manifests itself in the region of high frequencies ($\omega_0 \gg \omega_*$). Relation (6) here has the asymptotic

$$\omega = Q\omega_0 + \frac{1}{2}\,(1 - Q)\,(\omega_*^2/\omega_0)\,, \tag{7}$$

according to which rotation of a pair of particles ($Q = 0$) in the high-frequency region is attenuated as $1/\omega_0$ while the rotational frequency of a pair of drops grows proportionally with the frequency of the field. The dependence $\omega(\omega_0)$ is presented for several values of Q in Fig. 1.

Therefore, at the high-frequency limit when the comparatively slow processes of cross-linking are shut down, dissipative interaction of the emulsion with the field can be maintained by nonequilibrium processes within the drops.

We note that with a growth in the field frequency, when it becomes comparable with the inverse time of magnetization relaxation ($\omega_0\tau_r \approx 1$), there arises a need to allow for nonlinear effects of relaxation. This is also demonstrated by the results of the experiments [9]. In the final analysis, as $\omega_0 \to \infty$, the emulsion will lose its sensitivity to the field.

This investigation was financed by the Fundamental Research Fund of the Republic of Belarus.

NOTATION

t, time; **M**, magnetization; M_0, M_s, equilibrium magnetization and saturation magnetization; **H**, magnetic field strength; **h**, unit vector in direction of magnetic field; $L(*)$, Langevin function; ξ, Langevin parameter; m_0, magnetic moment of ferroparticle; m, magnetic moment of drop; k, Boltzmann constant; T, absolute temperature; τ_r, characteristic time of rotational diffusion; η_0, viscosity of carrier of magnetic liquid; η_r, rotational viscosity of magnetic liquid; η, viscosity of emulsion carrier; Ω, hydrodynamic vortex and rotational velocity of drop; ω_0, rotational frequency of field; ω_*, quench frequency of pair rotation that is synchronous with field; ω, average rotational velocity of pair; **T**, electrodynamic torque; **F**, dipole interaction force; **n**, unit vector of orientation for pair of drops.

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